Distance-Based Functional Diversity Measures and Their Decomposition: a Framework based on Hill Numbers

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Outline

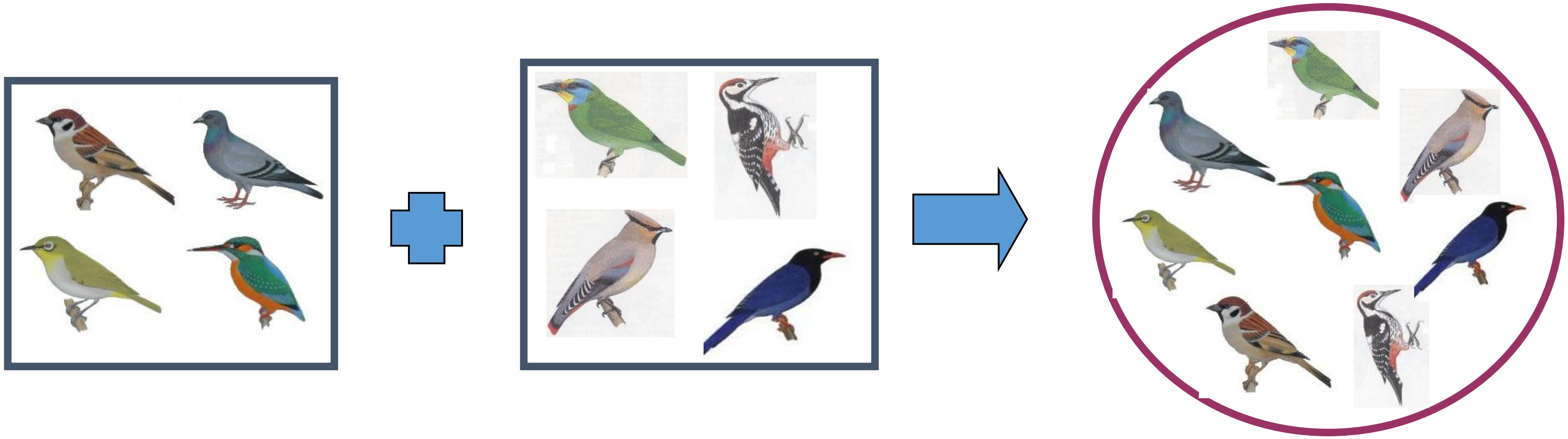
- Introduction
  - Hill numbers (1973 Hill)
  - Distance-based functional diversity indices
  - Motivation
- A framework base on Hill numbers
  - Functional diversity measures of a community
  - Functional differentiation among communities
- Example and Application
- Conclusions and discussions

Hill numbers

- Species diversity:
  - Species richness and Evenness among abundances
  - Monotonicity and Principle of Transfer (Patil & Taillie 1982)

- Diversity indices :
  - Richness
  - Shannon Entropy $-\sum_{i=1}^{S} p_i \log p_i$
  - Gini-Simpson index $1-\sum_{i=1} p_i^2$

- Replication principle (doubling property) (Hill 1973, Jost 2006)
  - Two completely distinct (no overlapped species) communities, each with diversity measure X
  - Combine these two, the diversity becomes 2X



- Species richness 4 + 4 = 8
- Entropy? 1.39 + 1.39 ≠ 2.08
- Gini_Simpson index? 0.75 + 0.75 ≠ 0.875

- Species richness 4 + 4 = 8
- Exp(entropy) 4 + 4 = 8
- Inverse (1-Gini_Simpson) 4 + 4 = 8

Hill Numbers (Hill 1973)

$$ {}^{q}D = \left(\sum_{i=1}^{S} p_i^q \right)^{1/(1-q)}, \quad q \geq 0 $$

- $q = 0$, ${}^{0}D$ =Species richness
- $q = 1$, ${}^{1}D$ = exponential of entropy $\quad {}^{1}D = \exp\left(-\sum_{i=1}^{S} p_i \log p_i \right)$
- $q = 2$, ${}^{2}D$ = inverse of Simpson index $\quad {}^{2}D = 1/\sum_{i=1}^{S} p_i^2$

- Effective number of species

Community1:
Hill numbers = D

=

Commumty2:
D species
with relative abundance:
1/D, …,1/D.

- Doubling property

The importance of "Doubling Property"

	1000 species →	600 species disappear	400 species survive
Gini-Simpson index	**0.999**	**0.9983**	**0.9975**

0.9983/0.999 = **0.999** almost diversity vanished

0.9975/0.999 = **0.998** almost diversity conserved

Inverse of Simpson index	1/(1-0.999) = 1000	1/(1-0.9983) = 600	1/(1-0.9975) = 400

Distance-based Functional Diversity

- Functional Attribute Diversity (FAD, Walker *et al.* 1999)

$$FAD = \sum_{i,j=1}^{S} d_{ij}$$

- Rao's quadratic entropy (Rao 1982)

$$Q = \sum_{i,j=1}^{S} p_i d_{ij} p_j \qquad \Rightarrow Q_\beta = \frac{Q_\gamma - Q_\alpha}{Q_\gamma}$$

- Effective number species with maximum distance (Ricotta & Szeill 2009, de Bello *et al.* 2010)

$$Q_e = \frac{1}{1 - Q / d_{\max}}$$

$$\Rightarrow Q^{*}_{e,\beta} = \frac{Q_{e,\gamma} / Q_{e,\alpha} - 1}{N - 1}$$

$$\Rightarrow Q^{**}_{e,\beta} = \frac{1 - Q_{e,\alpha} / Q_{e,\gamma}}{1 - 1 / N}$$

Motivation

- Example 1: two communities, each has *S* species and no shared species. The functional distance between two individuals is zero if they belong to the same species, and *1.* otherwise.

$$\Rightarrow Q_\beta = \frac{Q_\gamma - Q_\alpha}{Q_\gamma} = \frac{1}{2S-1} \qquad Q^*_{e,\beta} = \frac{Q_{e,\gamma}/Q_{e,\alpha} - 1}{N-1} = 1 \qquad Q^{**}_{e,\beta} = \frac{1 - Q_{e,\alpha}/Q_{e,\gamma}}{1 - 1/N} = 1$$

- Example2:

Region I

$$\begin{bmatrix} 0 & 0.1 & 0.2 & 0.2 \\ 0.1 & 0 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0 & 0.1 \\ 0.2 & 0.2 & 0.1 & 0 \end{bmatrix}$$

Region II

$$\begin{bmatrix} 0 & 0.1 & 0.2 & 0.9 \\ 0.1 & 0 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0 & 0.1 \\ 0.9 & 0.2 & 0.1 & 0 \end{bmatrix}$$

Measure	Case I		Case II
$Q_\beta = \frac{Q_\gamma - Q_\alpha}{Q_\gamma}$	0.091	<	0.310
$Q^*_{e,\beta} = \frac{1 - 1/Q_{e,\beta}}{1 - 1/N}$	0.250	>	0.127
$Q^{**}_{e,\beta} = \frac{Q_{e,\beta} - 1}{N-1}$	0.143	>	0.068

A framework based on Hill numbers

- Neutral diversity: $(p_1, p_2, \ldots, p_S) \Leftrightarrow (\frac{1}{D}, \frac{1}{D}, \ldots, \frac{1}{D})$

$$\sum_{i=1}^{S} 1 \times p_i^q = \sum_{i=1}^{D} 1 \times (\frac{1}{D})^q \Rightarrow {}^qD = \left(\sum_{i=1}^{S} p_i^q \right)^{1/(1-q)}$$

- Functional diversity:

$$\begin{bmatrix} d_{11} & d_{12} & \ldots & d_{1S} \\ d_{21} & d_{22} & \ldots & d_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ d_{S1} & d_{S2} & \ldots & d_{SS} \end{bmatrix} \begin{bmatrix} p_1^2 & p_1 p_2 & \ldots & p_1 p_S \\ p_2 p_1 & p_2^2 & \ldots & p_2 p_S \\ \vdots & \vdots & \ddots & \vdots \\ p_S p_1 & p_S p_2 & \ldots & p_S^2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 0 & Q^* & \ldots & Q^* \\ Q^* & 0 & \ldots & Q^* \\ \vdots & \vdots & \ddots & \vdots \\ Q^* & Q^* & \ldots & 0 \end{bmatrix} \begin{bmatrix} \left(\frac{1}{D}\right)^2 & \left(\frac{1}{D}\right)^2 & \ldots & \left(\frac{1}{D}\right)^2 \\ & & \ldots & \\ \vdots & \vdots & \ddots & \vdots \\ \left(\frac{1}{D}\right)^2 & \left(\frac{1}{D}\right)^2 & \ldots & \left(\frac{1}{D}\right)^2 \end{bmatrix}$$

, where $Q^* = QD/(D-1)$ and $Q = \sum_{i,j} p_i d_{ij} p_j$

$$\sum_{i=1}^{S}\sum_{j=1}^{S} d_{ij}(p_i\, p_j)^q = \sum_{i=1}^{D}\sum_{j=1}^{D} Q^*(\frac{1}{D^2})^q \Rightarrow {}^qD(Q) = \left[\sum_{i=1}^{S}\sum_{j=1}^{S} \frac{d_{ij}}{Q}(p_i\, p_j)^q \right]^{1/2(1-q)}$$

A framework based on Hill numbers

- Functional Hill number: $^{q}D(Q)=\left[\sum_{i=1}^{S}\sum_{j=1}^{S}\frac{d_{ij}}{Q}(p_i\, p_j)^q\right]^{\frac{1}{2(1-q)}}$
  - when d_{ij} is comstant, $^{q}D(Q)$ = ^{q}D (Hill 1973)

- (Total) functional diversity: $^{q}FD(Q)=[^{q}D(Q)]^2\times Q$
  - when q = 0, $^{0}FD(Q)$ = *FAD* (Walker et al. 1999)

- Mean functional diversity: $^{q}MD(Q){=}^{q}FAD(Q)/^{q}D(Q)$
  - when q = 0, $^{0}MD(Q)$ = *FAD/S* (*MFAD*, Schmera et al. 2009)

Partitioning Functional Diversity Measures

Functional Gamma diversity: $$ {}^{q}D_{\gamma}(Q)=\left[\sum_{i=1}^{S}\sum_{j=1}^{S}d_{ij}\left(\frac{p_{i+}p_{j+}}{Q}\right)^{q}\right]^{1/2(1-q)} $$

Functional Alpha diversity: $$ {}^{q}D_{\alpha}(Q)=\frac{1}{N}\left[\sum_{k,m=1}^{N}\sum_{i,j=1}^{S}d_{ij}\left(\frac{w_{k}p_{ik}w_{m}p_{jm}}{Q}\right)^{q}\right]^{1/2(1-q)} $$

Functional Beta diversity: $$ {}^{q}D_{\beta}(Q)=\frac{{}^{q}D_{\gamma}(Q)}{{}^{q}D_{\alpha}(Q)} $$

Functional Differentiation measures:

local differentiation: $$ 1-C_{qN}(Q)=\frac{[{}^{q}D_{\beta}(Q)]^{1-q}-1}{N^{(1-q)}-1} $$

regional differentiation: $$ 1-U_{qN}(Q)=\frac{[{}^{q}D_{\beta}(Q)]^{q-1}-1}{N^{(1-q)}-1} $$

Example

Region 1:

$$\begin{bmatrix} 0 & 0.1 & 0.2 & 0.2 \\ 0.1 & 0 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0 & 0.1 \\ 0.2 & 0.2 & 0.1 & 0 \end{bmatrix}$$

Region 2:

$$\begin{bmatrix} 0 & 0.1 & 0.2 & 0.9 \\ 0.1 & 0 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0 & 0.1 \\ 0.9 & 0.2 & 0.1 & 0 \end{bmatrix}$$

Measure	Order q	Case I		Case II
$1-C_{qN}(Q)$	$q=0$	**0.348**	<	**0.531**
	$q=1$	**0.364**		**0.517**
	$q=2$	**0.376**		**0.503**
$1-U_{qN}(Q)$	$q=0$	**0.516**	<	**0.694**
	$q=1$	**0.364**		**0.517**
	$q=2$	**0.231**		**0.336**
$Q_{\beta}^{*}=\frac{Q_{\gamma}-Q_{\alpha}}{Q_{\gamma}}$	$q=2$	**0.091**		**0.310**
$Q_{e,\beta}^{*}=\frac{1-1/Q_{e,\beta}}{1-1/N}$	$q=2$	**0.250**		**0.127**
$Q_{e,\beta}^{**}=\frac{Q_{e,\beta}-1}{N-1}$	$q=2$	**0.143**		**0.068**

Real data analysis (discussed by Ricotta et al. 2010)

Embryo (EM) dune

Mobile (MO) dune

Transition (TR) dune

EM (17 species)	MO (39 species)	TR (42 species)
16	16	16
1	1	
	22	22
		4

Diversity: TR > MO > EM

Real data analysis (discussed by Ricotta et al. 2010)

- 16 function traits (7 quantitative variables, 9 categorical variables)
- Using Gower distance to calculate functional distance

Measure	Order q	EM	MO	TR
Q	2	0.513	0.556	0.561
Q_e	2	2.94	3.39	2.95
${}^qFD(Q)$	0	162.15	902.18	1053.67
	1	50.78	247.68	351.30
	2	30.58	129.79	211.67
${}^qMD(Q)$	0	9.12	22.40	24.31
	1	5.10	11.74	14.04
	2	3.96	8.49	10.89
${}^qD(Q)$	0	17.77	40.26	43.33
	1	9.95	21.10	25.02
	2	7.72	15.27	19.42

Real data analysis (discussed by Ricotta et al. 2010)

Measure	Order	EM vs. MO	EM vs. TR	MO vs. TR
(Q_γ, Q_α)		(0.550, 0.535)	(0.561, 0.537)	(0.574, 0.559)
(1) Differentiation measure based on additively partitioning quadratic entropy				
$Q_\beta^* = \frac{Q_\gamma - Q_\alpha}{Q_\gamma}$	$q = 2$	0.0279	0.0421	0.0257
(2) Differentiation measure based on the effective number of maximally distinct species				
$Q_{e,\beta}^* = \frac{1 - 1/Q_{e,\beta}}{1 - 1/N}$	$q = 2$	0.0659	0.1021	0.0669
$Q_{e,\beta}^{**} = \frac{Q_{e,\beta} - 1}{N - 1}$	$q = 2$	0.0341	0.0538	0.0346
(3) Distance-based differentiation measures derived from the functional beta diversity				
$1 - C_{qN}(Q)$	$q = 0$	0.396	0.458	0.063
	$q = 1$	0.428	0.714	0.282
	$q = 2$	0.576	0.841	0.456
$1 - U_{qN}(Q)$	$q = 0$	0.567	0.628	0.118
	$q = 1$	0.428	0.714	0.282
	$q = 2$	0.405	0.725	0.295

Conclusions

- Rao's quadratic entropy and Ricotta's effective number of species with maximum distance may not be directly used to measure functional diversity and differentiation among communities.
- Extended ordinary Hill numbers to distance-based functional diversity measures (${}^qD(Q)$, ${}^qMD(Q)$, ${}^qFD(Q)$) to take into account the pairwise functional distance.
- We have developed the decomposition of proposed three functional diversity measures of any order q, where alpha and beta components are unrelated.
- Beta component measures can be transformed onto the range [0, 1] to obtain the normalized distance-differentiation measures.

Thank you very much.